Essays on Sovereign Debt Restructuring

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

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August, 2021

Acknowledgements

I am grateful to my advisors, Timothy Kehoe and Manuel Amador, for their guidance, support, and encouragement throughout the past 6 years. I am also especially thankful to Juan Pablo Nicolini and Marco Basseto for their attention and thoughtful advice. I also benefited from many fruitful conversations with the members of the Trade Workshop. My coauthors, Camilo Alvarez, Carlos Esquivel, Agustin Samano, and Pedro Tanure, also played important roles in shaping my economic thinking. The constant interaction with my office mates helped me focus on my studies and made the PhD journey very enjoyable. I was also lucky to find a great and supportive friend in Minneapolis, Meriem LeClair. And, of course, I'm thankful to my always present parents, Katia and Paulo, who have always given me the courage and strength I need for any challenge I happen to face.

Dedication

To my parents, Katia and Paulo, for their endless love, support, and encouragement.

Abstract

This dissertation consists of three chapters. In the first chapter, I provide a review of the literature on sovereign debt restructurings. I discuss clauses in sovereign debt contracts that often dictate the rules of debt restructuring. I also summarize some empirical regularities and policy concerns regarding debt relief, default duration and subsequent capital market exclusions, preemptive default, and legal disputes. Finally, I indicate how the literature on quantitative sovereign debt models has addressed these policy concerns.

In the second chapter, I develop a sovereign debt model with endogenous re-entry to international financial markets via debt renegotiation and a possibility for lenders to hold-out and litigate. This renders the outcome of a renegotiation process to be characterized by both a haircut and a lenders' participation rate. I use this model to show that the lenders' threat to litigate buys commitment to the sovereign. Precisely, in order to increase the lender's participation rate and hence reduce subsequent litigation, governments in default negotiate lower haircuts; as a result, lenders charge lower spreads ex-ante during the periods in which the country has access to international financial markets. I use this model to evaluate the role of collective action clauses and find that the optimal threshold for the economy of Argentina during the 1990s was 80%, which is only 5pp above the most widespread threshold used in sovereign debt contracts under NY law since the 2001 Argentine default.

In the third chapter, Agustin Samano and I study the optimal accumulation of international reserves in a two-period model of sovereign debt restructuring. I show that countries manage their reserves taking into account that these assets affect the outcomes of debt restructurings. In particular, countries may accumulate reserves while in default in order to improve their bargaining power in eventual debt restructurings.

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Chapter 1

Sovereign Debt Resolution: Literature Review

1.1 Introduction

Market incompleteness, government's lack of commitment, and limited enforcement mechanisms are three defining features of sovereign debt contracts. Throughout this chapter, I discuss some obstacles they impose to the design of sovereign debt contracts. In particular, I summarize important clauses in these contracts, present data on sovereign debt restructuring, and review modeling approaches in the literature of quantitative sovereign debt models.

Clauses in Sovereign Debt Contracts

The following clauses are commonly embedded in sovereign debt contracts and dictate the rules of restructuring processes:

- **Pari passu clauses** impose that bondholders rank equally among themselves and other unsecured obligations[1].

[1] And negative pledge clauses complements the pari passu by preventing the debtor from pledging any

- **Seniority clauses** allow bondholders to enjoy different treatment in the event of default. Senior debt is prioritized for repayment relative to junior debt.
- **Collective action clauses** define voting rules under which the debtor and a majority of bondholders can impose changes to the payment terms of debt contracts held by all bondholders, including the minority who votes against it.
- **Acceleration clauses** impose conditions under which bondholders may request immediate repayment of the full outstanding debt obligations.

Among other consequences, the government's inability to credibly constrain future borrowing decisions give rise to the debt dilution problem: future borrowing lead existing debt to face higher default probability[2]. This problem is reinforced by pari-passu clauses, which prevents current bondholders from enjoying preferential treatment in eventual debt restructurings.

In contrast, seniority clauses can mitigate this debt dilution. It may not fully eliminate this problem because further borrowing still increases a bond's default risk, irrespective of its seniority status, but seniority-dependent haircuts may reduce the burden that future borrowing imposes on senior debt.

Coordination problems also pervade sovereign debt restructurings. Collective action clauses are often introduced to diminish creditors' incentives to free ride on other creditors' debt relief who, otherwise, would have incentives to hold out of renegotiations. These clauses impose rules under which a minority of bondholders are forced to accept the changes in repayment terms negotiated between the government and a majority of bondholders.

I leave the discussion on acceleration clauses to section 1.2, where this clause motivates one of the definitions and measurements of debt relief. Throughout this chapter, I highlight how empirical and quantitative studies have analyzed these clauses[3].

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assets.

[2]See Chatterjee and Eyigungor (2015) for an application of this argument in sovereign debt markets and Bizer and DeMarzo (1992) for a more general discussion.

[3]Choi et al. (2011) provide a comprehensive discussion on the evolution of sovereign debt clauses over the last decades.

1.2 Empirical Work

There are important empirical regularities that motivate the study of sovereign debt restructuring. In this section, I summarize regularities regarding debt relief, default duration and subsequent capital market exclusions, preemptive default, and legal disputes.

1.2.1 Debt Relief

Debt relief is characterized by a reduction in the stream of due payments or an extension of their due date. Cruces and Trebesch (2013) provide a wide dataset that covers three alternative approaches to calculate the lenders' losses involved in debt restructuring episodes. Nevertheless, these approaches only imperfectly capture different debt relief dimensions.

For instance, as the name suggests, face value haircuts takes into account only one dimension of a lender's loss. In particular, it ignores any maturity extension associated with the restructuring. Cruces and Trebesch (2013) illustrate this measurement's limitation: since the 1970s, there have been at least 123 episodes in which the debt relief involved no face value reduction, but debt maturity lenghtening.

Two more widely used approaches compare the present value of the new/restructured debt (plus cash received) against two alternative measures of the old debt. One of them relies on the face value of the old outstanding debt, while the other one relies on its present value.

$$\text{Haircut} = \frac{\text{Present value of new debt}}{\text{Face or present value of old debt}} \tag{1.1}$$

Using the old debt's face value can be justified by the widespread presence of acceleration clauses in sovereign debt contracts. These clauses impose conditions under which the loan payment is anticipated, becoming fully and immediately due. In sovereign debt contracts, default status is often a condition that activates these clauses.

Alternatively, using the old debt's present value, first proposed by Sturzenegger and Zettelmeyer (2006), is a more rigorous approach and has become the preferred one among academics. Nevertheless, it still has its drawback. Present values, both of the new and old debt, are discounted using the yield at the exit of a default episode[4]. Therefore, renegotiation outcomes affect the measurement of the lenders' losses not only by affecting the new but also the old debt level. For instance, if the debt forgiveness is very generous to the government, it causes a considerable improvement to the country's debt sustainability, which reduces the yield; consqueqently, the measurement of the old debt increases.

Using this later debt relief measurement, Cruces and Trebesch (2013) show that the average haircut on sovereign debt is 37% and that it has increased over the recent decades[5].

Dvorkin et al. (2021) uses the dataset of Cruces and Trebesch (2013) to infer maturity extensions. They find that the majority of these restructuring events involved some maturity extension, and their average extension was 3.4 years. They also show that countries received longer extensions the higher their output recovery.

1.2.2 Capital Market Exclusion and Default Duration

Default costs take different forms. A distinctive and quantifiable one regarding debt restructuring is exclusion from international financial markets. Its duration is defined as the time interval between a successful renegotiation and international financial market reaccess[6].

The main take away from Cruces and Trebesch (2013) and Richmond and Dias (2009) is the following. Haircuts and international financial market exclusion are strongly correlated[7]. Natural disasters seem to provide reasonable excuse for a shorter market exclusion.

[4]Sturzenegger and Zettelmeyer (2006) and Sturzenegger and Zettelmeyer (2008) compute the implicit yields from secondary markets. To extend the sample beyond the countries with liquid debt, Cruces and Trebesch (2013) discount the expected cash flows using an inputed yield based on country-specific and contemporaneous credit risk premium

[5]Haircuts on sovereign debt are lower than haircuts on corporate debt. It's important to notice that governments borrow in an environment with weaker enforcement mechanisms than firms. Thus, it's natural that corporate defaults more often (than sovereign defaults) take place in states in which the debtor cannot (as opposed to "choose not to") repay creditors.

[6]See Richmond and Dias (2009), Gelos et al. (2011), and Cruces and Trebesch (2013).

[7]Relatedly, Cruces and Trebesch (2013) show that haircuts are positively correlated with another cost: post-restructuring spreads.

The average market exclusion lasts around 5 years (the median is around 3 years).

And notice that the period in which a country stays in financial autarky (i.e., with limited or no access to international financial markets) is longer. It actually starts at the time of default. And Benjamin and Wright (2013) show that renegotiation are protracted and take an average of 8 years to resolve[8].

1.2.3 Preemptive default

While default can often be protracted and impose a long period of international financial market exclusion, preemptive restructuring is also frequent. Asonuma and Trebesch (2016) indicate that, from 1978 to 2010, 38% of debt restructurings were preemptive. They also indicate that these episodes tend to be more quickly resolved and feature lower haircuts and output losses.

1.2.4 Legal Disputes

Schumacher et al. (2018) provides a comprehensive dataset on events of sovereign debt litigation starting in 1970. Their main takeaway is the following. Attempts to enforce sovereign debt payments through litigation are becoming more prevalent: debt under dispute has increased from near 0% in the 1980s/early 1990s to 1.5% of debtor countries' GDP in the 2000s. Countries tend to experience longer international financial market exclusion when they face these lawsuits: it reduces the probability of reaccessing international financial markets by 16%. A small number of plaintiffs, usually distressed debt funds, initiate most of the lawsuits: for instance, the 12 largest litigating creditors hold more than twice as much the remaining litigating creditors. Moreover, out of 109 lawsuit cases with available information, only a small portion were resolved on court: 10 cases in favor and 14 against creditor claims; the remaining majority was settled out of court.

There is also a large empirical literature on the pricing implications of including CACs in bond contracts. Besides pointing out a positive correlation between holdout rates and

[8]Benjamin and Wright (2013) show that, as opposed to default that tends to occur when the country's output is below trend, renegotiation tends to conclude when the output has returned to trend.

haircuts at the bond level, Fang et al. (2021) also highlight the effect of CACs on sovereign debt markets: CACs decrease holdout rates by half, and their voting thresholds are rarely unachieved.

Carletti et al. (2020) and Chung and Papaioannou (2020) further explore CACs and holdout activity. They find that bonds trade at lower yields in secondary markets when CACs are embedded in the contract, especially for countries with worse credit ratings.

1.3 Quantitative Work

In this section, I discuss alternative approaches to modeling renegotiation and significant contributions, mostly from the literature of quantitative sovereign debt models, related to the empirical work from section 1.2. The policy implications from most of these models are useful for evaluating the consequences of clauses in sovereign debt contracts and, ultimately, the design of these contracts.

1.3.1 Bargaining Process

While the original quantitative papers assumed that, after default, countries eventually reacess international financial markets with zero debt, an increasing number of studies have modeled the bargaining process so that outcomes are endogenously determined.

The pioneer in this field was Yue (2010), in which the the parties negotiate a haircut through a Nash bargaining process. The government negotiates with lenders, who act as a coordinated group, and they jointly maximize their surpluses.

Even though Nash bargaining has been the most widely employed cooperative bargaining in the literature, it is not the only one. Wang (2019) relies on Kalai-Smorodinsky bargaining. These approaches depart from common axioms: Pareto optimality, symmetry, and scale-invariance. Nevertheless, the Kalai-Smorodinsky bargaining drops the independence of irrelevant alternatives axiom that is present in the Nash bargainig, and replace it with a resource monotonicity axiom. This axiom implies that the lenders' recovery is higher the larger is the stock of defaulted debt. Thus, there is a sense of "fairness" in the Kalai-Smorodinsky bargaining solution. Wang (2019) argue that this might be a desirable feature, given that debt restructurings are usually intermediated by organizations like the IMF. Moreover, they point out that, as opposed to Nash bargaining, this approach generates a positive correlation between debt recovery and defaulted debt[9].

On the non-cooperative bargaining side, Pitchford and Wright (2007) and Bi (2008) introduce a modified version of Rubinstein (1982) to the literature of quantitative sovereign debt models in which the parties make alternating offers. Their modeling decision allows their models to generate endogenous renegotiation delays in equilibrium. This would not be feasible with Nash bargaining, as it relies on an exogenous parameter (the probability that a renegotiation opportunity exogenously arises) for generating delays. Still, Pitchford and Wright (2007) and Bi (2008) have different mechanisms behind delays. In the former, it stems from creditors free-riding on renegotiation efforts of others; in the latter, it stems from the parties waiting for an economic recovery before they settle an agreement.

1.3.2 Selected papers

Face value haircuts were the only renegotiation outcome in the first sovereign debt models. Mihalache (2020) develops a framework that also captures maturity extensions. He only considers two borrowing instruments, one short term and the other one long term, where the share of each instrument determines the average maturity of the debt stock. Dvorkin et al. (2021), on the other hand, considers a setting with a richer set of borrowing instruments, where the goverment can issue bonds of many different maturities[10].

In a similar framework that allows for a rich set of borrowing instruments, Dvorkin et al. (2019) evaluate the cost of debt dilution and evaluate different policy interventions. In particular, they show that tilting creditor losses towards short maturity debt may reduce the short term yield spreads and maturity during financially distressed times, which in turn

[9]Notice, though, that alternative modeling decisions are likely to generate this same positive correlation between debt recovery and defaulted debt. For instance, if partial default is an available option (in which the country continues to service a portion of its debt), then the value in default may depend on the debt level, and so may the outcome of renegotiation.

[10]Note that Mihalache (2020) uses Nash bargaining while Dvorkin et al. (2021) follows Benjamin and Wright (2013).

reduces the default probability and portfolio's borrowing costs. They also show that GDP-indexed bonds could help facilitate market access and reduce the probability of repeated restructurings.

Chatterjee and Eyigungor (2015) study debt dilution problems and seniority arrangements. In particular, they consider a seniority arrangement promoted by Bolton and Skeel Jr (2005) in which, after a default, proceeds from the settlement are distributed in the order of absolute priority, which means that older debt is paid before newer debt. They find that, in the optimal arrangement, only a share of each bond is protected by a seniority clause. This setup mitigates both debt dilution and conflicts between creditors regarding the size of the haircut. Therefore, their proposal includes features of the Pari-Passu and seniority clauses.

Interest rate shocks also affect debt restructuring outcomes. Almeida et al. (2019) introduce a new channel through which increases in the risk-free interest rates can trigger sovereign debt crisis: debtor-countries anticipate that they can bargain better restructuring terms in higher risk-free interest rate environments. Thus, such increases not only raise the cost of repayment (by making debt rollover more expensive), but also make default less costly (by making the debt relief more generous). They apply their framework to shed some light on the 1980s Lost Decade in Latin America and find that the Federal Reserve's policy choice of raising interest rates to tame US inflation was a likely trigger of the 1982 default in Mexico. Interestingly, they show that without a bargaining process, the model cannot account for any relevant relationship between the risk-free interest rate and default decision.

Tavares (2015) studies the role of international reserves in sovereign debt restructuring under fiscal adjustment. In his framework, reserves bring insurance benefits against default and, especially, rollover risk. In particular, reserves reduce the need for distortionary taxation. He also shows that reserves increase the debt recovery rate, which then reduces the borrowing costs.

Credit default swaps (CDS) are another instrument available in sovereign debt markets that provides insurance benefits. Creditors can insure against default events by purchasing a CDS, in which the seller of the CDS compensates the creditor in case of default. Salomao

(2017) shows that this instrument improves the creditors' bargaing position and, hence, leads to higher recovery rates. As opposed to foreign reserves that protect governments against default, CDS protects creditors. Yet, since default becomes more costly, governments still enjoy benefits from the existence of CDS in the form of higher commitment to repayment, and ultimately lower default probability and borrowing costs.

In chapter 2, I present a framework to study the optimal threshold for collective action clauses. I show that borrowing is cheaper due to the threat of litigation, and hence CACs thresholds should not be too low to the point of eliminating this beneficial effect. On the other hand, CACs facilitate orderly restructurings by preventing that a minority of creditors disrupt a previous deal and force the government back to financial autarky. Thus, the introduction of CACs provide a balance between the ex-ante extra commitment for borrowing that stems from litigation and its associated post-restructuring (higher borrowing) costs.

Hatchondo et al. (2014) show that voluntary debt exchanges can be detrimental to the economy, even if both parties are better off at the time of restructuring. This result indicates that initiatives that facilitate sovereign debt restructurings may be undesirable for long-term outcomes.

1.4 Conclusion

In this chapter, I review some data pattern on sovereign debt contracts and how the literature on quantitative sovereign debt models has evaluated common problems that arise from sovereign debt renegotiation. These problems have clear policy implications as they affect the countries' default incentives and borrowing costs.

Chapter 2

The Holdout Problem in Sovereign Debt Markets

2.1 Introduction

Sovereign debt markets feature a holdout problem: in any debt restructuring episode, each creditor has incentives to free-ride on the debt relief provided by the other creditors. Instead of accepting the deal as its peers, it may engage in a litigious process in an attempt to obtain a higher recovery rate from the then less financially-distressed government.

This process, which jeopardizes the post-restructuring recovery of debtor countries, has become widespread in recent decades. Schumacher et al. (2018) document that the litigated claims in a US or UK court as a share of the debtor countries' GDP has risen from 0.4% in the 1990s to 1.6% in the 2000s.

The escalation of litigation led the international community to look for ways to minimize this holdout problem, and the most prevalent solution involves a contractual approach: collective action clauses (CACs) have been embedded in new bond issuances to prevent the emergence of holdouts[1]. In general, these clauses allow a majority to impose restructuring

[1] The international community has also looked for approaches other than CACs. In Belgium, a recently enacted law limits the creditors' ability, under certain circumstances, to recover through litigation more

terms on a minority of bondholders. Bradley and Gulati (2014) document a shift towards CACs in 2003: 95% of sovereign bonds issued in New York required unanimity in the decade preceding this date, while virtually none in the subsequent one. Likewise, in response to the European sovereign debt crisis, all countries in the euro area are required to include CACs in their new sovereign bonds since 2013.

The purpose of this paper is to provide a default framework à la Eaton and Gersovitz (1981) to evaluate the effect of litigation in sovereign debt markets and the design of sovereign debt contracts. I introduce a theory of debt restructuring where identical risk-neutral foreign lenders make individual decisions on whether to accept the restructuring terms or to engage in litigation. When litigation succeeds, the government is forced to either default on all bondholders or fully repay the holdouts. Thus, the model features an endogenous lenders' participation rate that helps discipline the debt relief through two channels. The first is very direct: high haircuts make the deal less attractive to lenders and induce low participation rates. The second one regards the value of a bond in legal disupute: when litigation succeeds, the government is more likely to fully repay holdouts and avoid a new default event if there is little holdout debt; thus, high participation rates require low enough haircuts to offset the free-riding incentives. Therefore, the threat to litigate enhances the government's commitment to repay its debt. As a consequence, litigation reduces sovereign spreads when the government is in good financial standing. Nevertheless, new borrowing becomes more expensive under the presence of holdouts, as they increase the default risk.

The introduction of CACs provide a balance between the ex-ante extra commitment for borrowing that stems from litigation and its associated post-restructuring (higher borrowing) costs. All agreements that lead to participation rates below the CAC threshold still benefit from the threat to litigate, ensuring lower haircuts for those lenders that participate in the deal. Yet, CACs prevent small shares of lenders from free-riding on the debt

than the price they paid for the bonds. In 2018, the European Parliament incentivized member states to adopt similar regulations. The UK protects the Heavily Indebted Poor Countries (HIPC), as litigation cannot render more favorable terms than those agreed under the HIPC Initiative. Besides the "anti-vulture fund" legislation, the IMF proposed the Sovereign Debt Restructuring Mechanism (SDRM), which was rejected because of the lack of support from the US; see Krueger (2002).

relief, thus minimizing the coordination problem between participating lenders, holdouts, and future lenders.

I calibrate my model using data from Argentina for the period preceding its 2001 default episode and find that the optimal CAC has an 80% threshold, which is only 5pp above the typical threshold currently used in sovereign debt contracts under NY law[2]. This 2001 event is one of the last default episodes before CACs became prevalent under NY law and illustrates how holdouts can disrupt the restructuring process when these clauses are not present. After the 2001 default and two rounds of restructuring in 2005 and 2010, Argentina modified the payment terms of 93% of its bonds with a 70% haircut. The holdouts, who represented 7% of the original stock of debt in default, got several favorable judgments during the 2000s that established that they were entitled to full face value rather than, for instance, the price for which they purchased the bonds or the value that other lenders settled in 2005 and 2010. Despite the barriers to seizing Argentine assets due to sovereign immunity, holdouts won an important injunction in 2012 that forced Argentina to either default on all lenders or restructure the bonds held by the holdouts[3].

The paper proceeds as follows. I briefly overview the related literature in the remainder of this introduction. In Section 2.2, I describe the model and, in Section 2.3, I inspect its mechanisms. Then, in Section 2.4, I calibrate the model and present the numerical results. Finally, I conclude the paper in Section 2.5.

My paper is connected to the quantitative literature that follows Eaton and Gersovitz (1981) that had its early quantitative applications with Arellano (2008) and Aguiar and Gopinath (2006) in a setting with zero recovery rate on defaulted debt. Subsequently, Yue (2010) introduces renegotiation to standard sovereign default models using cooperative game theory solution concepts. Like Hatchondo et al. (2014), Mihalache (2020), and

[2]Bradley and Gulati (2014) report that CAC thresholds range from 18.75% to 85%. The 18.75%, though, usually applies only when an initial quorum requirement is not satisfied. And the most common threshold is 75%.

[3]The US courts had jurisdiction to issue an injunction relief because the contracts required payments to be made through a trustee. Thus, until Argentina settled an agreement with holdouts, the trustee could not realize the payments to those bondholders who had previously agreed to a haircut in 2005 and 2010. For further details on the Argentina negotiations, see Alfaro (2015).

Almeida et al. (2019), I follow Yue (2010) in the particular aspect of modeling debt restructuring as the outcome of a Nash bargaining problem between the government and the (participating) lenders. Nevertheless, the participation rate can be smaller than 100% in my model. This potential lack of cooperation, with non-participating lenders free-riding in the participating ones' debt relief, is exactly what gives rise to the holdout problem.

Benjamin and Wright (2013) introduce renegotiation using non-cooperative game theory solution-concepts to the sovereign default literature. My paper is mostly related to Pitchford and Wright (2012) paper about the holdout problem. They use a non-cooperative approach to quantitatively analyze delays in debt restructurings. An essential difference between our papers regards the rounds of renegotiations and the sources of inefficiency. They assume that the government negotiates with one bondholder per time, and each defaulted bond guarantees its holder a veto power over the country's ability to reaccess international financial markets. Then, it creates incentives for each bondholder to be the last one to restructure the debt and, consequently, causes inefficiencies through delays. In contrast, in my paper, there is only one round of renegotiation: the restructuring offer is simultaneously available to all bondholders, who can reject it, hold out, and litigate. Here, the inefficiency stems from the higher borrowing costs the government faces while dealing with holdouts.

Closest to my paper is Anand and Gai (2019), who develop an analytical framework for sovereign debt negotiations with endogenous participation rates. In their setting, though, the government tailors the bankruptcy procedures by committing in advance to a haircut in the event it files for bankruptcy and seeks restructuring. In contrast, in my setting, the government lacks commitment regarding the haircut. Bi et al. (2016) also develop a simple and elegant framework to thoroughly discuss different aspects of sovereign debt contracts. Neither of these papers, though, provide a quantitative exercise.

There is a large empirical literature on the pricing implications of CACs in bond contracts. The study that uses the most comprehensive data is Chung and Papaioannou (2020), which finds evidence that the inclusion of CACs is associated with lower borrowing costs. At first glance, this finding may seem conflicting with my model's results, in which CACs weaken the extra borrowing commitment that litigation provides and hence leads to higher borrowing costs. Nevertheless, there is a critical difference between the object of analysis of Chung and Papaioannou (2020) and mine. I compare the interest rates on the Argentine bonds of the 1990s, absent of CACs, against a counterfactual in which CACs are embedded in all Argentine bonds. On the other hand, Chung and Papaioannou (2020) consider a setting where countries simultaneously hold both types of bonds; in most years of their panel data, the countries' outstanding debt was composed of bonds with and without CACs. This difference has important implications: while the full replacement of bonds with no CACs by bonds with CACs reduce the commitment to repay, the partial replacement may simply allow the holders of bonds with no CACs to free-ride on the holders of bonds with CACs. The reason is that renegotiation is more likely to orderly succeed when CACs are embedded in the contracts and, therefore, holders of bonds with CACs are the most likely ones to provide debt relief[4]. In a sense, the empirical analysis of Chung and Papaioannou (2020) evaluates the effect of CACs on spreads during a slow transition period while my quantitative model evaluates an immediate transition.

My paper also complements the empirical literature on sovereign debt restructuring. Schumacher et al. (2018) provide many empirical regularities and a comprehensive discussion on relevant institutional changes that shaped sovereign debt markets and particularly litigation processes. Fang et al. (2021) is even closer to my paper. They document, for instance, a positive correlation between the haircut and the holdout rate. I endogeneize this feature in my model and argue that it is a consequence of the government's consumption smoothing motive.

2.2 Model

I consider a small open economy à la Eaton and Gersovitz (1981) in which the government receives a stochastic endowment stream and issues non-state-contingent defaultable bonds to a large number of risk-neutral foreign lenders. Whenever the government defaults, it suffers a direct output cost and stays in financial autarky until the debt is restructured. A key feature is that each lender makes its individual decision on whether to accept or

[4]Bolton and Jeanne (2009) explain the replacement of bank loans with bonds using a similar reasoning.

reject the restructuring terms, subject to the collective action clause, which renders the participation rate endogenously. Afterward, the holdouts immediately engage in litigation against the sovereign, which eventually forces the government to either fully repay them or default on the entire stock of debt.

2.2.1 Government

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. In each period, households receive a stochastic endowment of a tradable good y_t that follows a finite-state Markov chain with transition probabilities $Prob\left(y_{t+1} = y'|y_t = y\right) = F\left(y'|y\right)$. The economy is populated by identical households, whose preferences are given by:

$$E_t \left[\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right) \right] \tag{2.1}$$

where $\beta \in (0, 1)$ is the subjective discount factor, c is consumption, and the utility function u is strictly increasing and strictly concave. The government is benevolent and can borrow from foreign lenders by issuing one-period non-contingent bonds.

Every period, the government observes the total stock of debt b, the stock of debt held by holdouts b_l, the income shock y, and whether it has access to international financial markets, where $z = 1$ indicates it does while $z = 0$ indicates it is in financial autarky. I assume $b \in \mathcal{B} = \left[0, \bar{b}\right]$ where $\bar{b} > 0$ is finite, so that the government cannot run a Ponzi scheme. Because the government savings are risk-free and the debt held by holdouts is, by construction, smaller than the total stock of debt, then $b_l \in [0, b]$. I also assume the government cannot come to any agreement with the holdouts through means other than litigation. In the periods in which the government does not inherit a previous default decision, litigation succeeds with an exogenous probability θ_L.

In case litigation fails, the government chooses between default and repayment. In this case, the value function of the government is:

$$V\left(b, b_l, y\right) = \max_{d \in \{0,1\}} \left\{ dV^D\left(b, y\right) + (1-d)\, V^P\left(b, b_d, y, 1\right) \right\} \tag{2.2}$$

where V^D is the default value, V^P is the repayment value, and default and repayment decisions are represented by $d = 1$ and $d = 0$, respectively.

In case litigation succeeds, the government must fully repay the holdouts if it chooses to avoid a default episode. Therefore, litigation can have negative consequences to non-holdouts. Then, the value function is:

$$V^L(b, b_l, y) = \max_{d_L \in \{0,1\}} \left\{ d_L V^D(b, y) + (1 - d_L) V^P(b, 0, y, 1) \right\} \tag{2.3}$$

where default and repayment decisions are represented by $d_L = 1$ and $d_L = 0$, respectively.

The value function when the government has access to international financial markets and chooses to repay (non-holdout) lenders is the following:

$$\begin{aligned} V^P(b, b_l, y, 1) &= \max_{b'} \left\{ u(c) + \beta \mathbb{E}_{y'|y} \left[(1 - \theta_L) V\left(b', b_l, y'\right) + \theta_L V^L\left(b', b_l, y'\right) \right] \right\} \\ s.t. \quad c &= y - (b - b_l) + q^P\left(b', b_l, y\right)\left(b' - b_l\right) \end{aligned} \tag{2.4}$$

The government can finance its consumption with its income y and new debt issue $(b' - b_l)$ at price $q^P(b', b_l, y)$, net of the debt service $(b - b_l)$. Notice the country still faces litigation risk in the subsequent period.

The case in which the government repays its debt but has no access to international financial markets is slightly different than the previous one. The value function is the following:

$$\begin{aligned} V^P(b, b_l, y, 0) &= u(c) + \beta \mathbb{E}_{y'|y} \left[(1 - \theta_L) V\left(b_l, b_l, y'\right) + \theta_L V^L\left(b_l, b_l, y'\right) \right] \\ s.t. \quad c &= y - (b - b_l) \end{aligned} \tag{2.5}$$

Notice that I assume the government faces litigation even if there is a measure zero of holdouts, $b_l = 0$. Besides avoiding a separate definition of another value function $V^P(b, 0, y, z)$, this approach is consistent with a price schedule in which a measure zero of holdouts still litigate and recover the full payment on their claims[5]. Furthermore, since restructuring a measure zero of debt have no impact in the government's payoff, i.e., $V(b', b_l, y') = V^L(b', b_l, y')$ when $b_l = 0$, then the alternative value function $V^P(b, 0, y, z)$ would be isomorphic to the one I use.

When the government chooses to default, it is excluded from international financial markets, suffers a direct output cost, $\phi(y)$, which is increasing in income, y, its debt service is suspended, and its stock of debt is frozen and carried to the next period. The associated value function is:

$$V^D(b, y) = u(y - \phi(y)) + \beta \mathbb{E}_{y'|y} \left[\theta_R V^R(b, y') + (1 - \theta_R) V^D(b, y') \right] \tag{2.6}$$

Notice that, after inheriting a previous default decision, renegotiation opportunities arise with an exogenous probability θ_R. When these opportunities arise, the outcome that follows the bargaining process is characterized by a participation rate and a haircut. The government can choose to accept the newly restructured debt level or remain in default. The restructuring immediately ceases the direct output cost $\phi(y)$, but the government remains in financial autarky in the current period, i.e., $z = 0$, and only regains access to financial markets in the subsequent one[6]. The associated value function is:

$$V^R(b, y) = \max_{d_R \in \{0,1\}} \left\{ d_R V^D(b, y) + (1 - d_R) V^P\left(b^R(b, y), b_l^R(b, y), y, 0\right) \right\} \tag{2.7}$$

where $d_R = 0$ if the government takes the deal with lenders and $d_R = 1$ otherwise, $b^R(b, y) \equiv PR^R(b, y)\left[1 - h^R(b, y)\right] b + \left[1 - PR^R(b, y)\right] b$ is the new total stock of debt after the debt restructuring, $b_l^R(b, y) \equiv \left[1 - PR^R(b, y)\right] b$ is the part of it that is held

[5]It recovers the full payment on their claims unless the government is in a state in which it would default regardless of the litigation success.

[6]This assumption of remaining in financial autarky in the period of the debt restructuring serves a computational purpose only. After the debt restructuing, the government does not make any immediate borrowing decisions, which simplifies the pricing equation (2.19). It has the benefit of reducing the jumps associated to the mapping of the prices that takes place in each iteration. For alternative solutions to this computational obstacle, see Chatterjee and Eyigungor (2012) and Gordon (2019).

by holdouts. The haircut that non-holdout lenders provide is given by $h^R(b,y)$ and the participation rate is given by $PR^R(b,y)$. In sections 2.2.2 and 2.2.3, I discuss in detail how $h^R(b,y)$ and $PR^R(b,y)$ are determined.

The solution to the government's problem gives decision rules for consumption, $c^P(b,b_l,y)$, debt issuance, $\left[b^P(b,b_l,y)-b_l\right]$, default policies, $d^P(b,b_l,y)$ and $d_L^P(b,b_l,y)$, and restructuring policies, $d_R^P(b,y)$.

2.2.2 Renegotiation

Following a default episode, renegotiation opportunities arise with probability θ_R. In this case, the government and the lenders negotiate a haircut $\tilde{h}$ after observing the amount of debt in default b and the income shock y. Thus, they face the following Nash bargaining problem.

$$\begin{aligned} h^R(b,y) &= \arg\max_{\tilde{h}\in[0,1]} \left\{ S^{LEN}\left(\tilde{h},b,y\right)^{\alpha} S^{GOV}\left(\tilde{h},b,y\right)^{1-\alpha} \right\} \\ s.t. &\quad S^{LEN}\left(\tilde{h},b,y\right), \quad S^{GOV}\left(\tilde{h},b,y\right) \geq 0 \end{aligned} \tag{2.8}$$

where α is the bargaining power of the participating foreign lenders, S^{LEN} is their surplus, and S^{GOV} is the government's. As usual, a constraint to this problem is that all parties need to be better off with the terms of renegotiation, otherwise it fails.

The participating lenders' surplus is the difference between resuming debt payments with a haircut $\tilde{h}$ and the market value of their bonds in case the government remains in default:

$$S^{LEN}\left(\tilde{h},b,y\right) \equiv \left(1-\tilde{h}\right)\tilde{PR}_{CAC}^{R}\left(\tilde{h},b,y\right)b - q^D(b,y)\,\tilde{PR}_{CAC}^{R}\left(\tilde{h},b,y\right)b \geq 0 \tag{2.9}$$

where $\tilde{PR}_{CAC}^{R}\left(\tilde{h},b,y\right)$ is the endogenous participation rate associated with a restructuing offer $\tilde{h}$, and $q^D(b,y)$ is the price schedule in secondary markets of a unit of a bond in default.

And I define the surplus of the govenment in an analogous way: it's the difference between the value of accepting the deal and the value of remaining in default, which happens if renegotiation fails:

$$S^{GOV}\left(\tilde{h}, b, y\right) \equiv V^{P}\left(b^{R}\left(\tilde{h}, b, y\right), b_{l}^{R}\left(\tilde{h}, b, y\right), y, 0\right) - V^{D}\left(b_{d}, y\right) \geq 0 \tag{2.10}$$

where $b^{R}\left(\tilde{h}, b, y\right) \equiv \left(1-\tilde{h}\right) \tilde{PR}_{CAC}^{R}\left(\tilde{h}, b, y\right) b + \left[1-\tilde{PR}_{CAC}^{R}\left(\tilde{h}, b, y\right)\right] b$ is the new total stock of debt after the debt restructuring and $b_{l}^{R}\left(\tilde{h}, b, y\right) \equiv \left[1-\tilde{PR}_{CAC}^{R}\left(\tilde{h}, b, y\right)\right] b$ is the part of it that is held by holdouts.

Finally, the outcome of this bargaining game is not only a haircut $h^{R}(b, y)$ but also a participation rate $PR^{R}(b, y) \equiv \tilde{PR}_{CAC}^{R}\left(h^{R}(b, y), b, y\right)$, i.e., the endogenous participation rate mentioned above, evaluated at the new restructured debt level. In the next section, I discuss how the participation rate for different restructuring offers is determined.

2.2.3 Lenders

There is a continuum of atomistic lenders indexed by i. Given their size, no lender can individually affect the participation rate. I assume the lenders' coalition that participates in the Nash bargaining process stems from their individual decisions. Thus, facing an offer $\tilde{h}$ and taking as given the participation rate $\tilde{PR}$, the problem of lender i in a restructuring episode is to choose whether to take the deal or to hold out:

$$a_{i}^{P}\left(\tilde{h}, \tilde{PR}, b, y\right) \in \arg\max_{a_{i} \in \{0,1\}} \left\{a_{i} RR\left(\tilde{h}\right) + \left(1-a_{i}\right) HO\left(\tilde{h}, \tilde{PR}, b, y\right)\right\} \tag{2.11}$$

where $a_{i}=1$ if the government takes the deal and receives as payoff a recovery rate RR on the unit of debt, and $a_{i}=0$ if the government rejects the deal and receives the payoff HO associated with the expected future gains from litigation.

The recovery rate is defined as one minus the haircut:

$$RR\left(\tilde{h}\right) \equiv 1-\tilde{h} \tag{2.12}$$

I define the value of the holdout strategy as:

$$HO\left(\tilde{h}, \tilde{PR}, b, y\right) \equiv \left(1-\epsilon_{CAC}\right) HO_T\left(\tilde{h}, \tilde{PR}, b, y\right) + \epsilon_{CAC} HO_1\left(\tilde{h}, \tilde{PR}, b, y\right) \tag{2.13}$$

I assume lenders believe the CAC is neglected with probability $\epsilon_{CAC} > 0$ close to zero. When it happens, the debt contract requires unanimity to implement changes in the payment terms. The terms HO_1 and HO_T indicate the value of a contract when unanimity is required and when the CAC threshold T is observed, respectively. They are defined as follows:

$$HO_t\left(\tilde{h}, \tilde{PR}, b, y\right) \equiv \begin{cases} q^L\left(\left[1-\tilde{PR}\right]b, \left[1-\tilde{PR}\right]b, y\right) & \text{if } \tilde{PR} < t \\ RR\left(\tilde{h}\right) & \text{otherwise} \end{cases} \tag{2.14}$$

where $q^L\left(\left[1-\tilde{PR}\right]b, \left[1-\tilde{PR}\right]b, y\right)$ is the price schedule of a bond held by a holdout after the government restructures the debt of lenders who participate in the deal. The payoff when $\tilde{PR} \geq T$ captures the lenders' compliance with the CAC: when a large enough majority of bondholders agree to a particular change in the payment terms, such changes are imposed to the minority of bondholders that otherwise would hold out and litigate.

Given that the disregard for CACs occurs with a negligible probability ϵ_{CAC}, the value of holding out is well captured by $HO_t\left(\tilde{h}, \tilde{PR}, b, y\right)$. The introduction of CAC negligence serves only to eliminate an undesirable equilibrium outcome: if CACs were always enforceable, then lenders would be indifferent between accepting or rejecting the deal independently of the haircut offer whenever they take as given a 100% participation rate. In section 2.3, I explore in detail the equilibrium selection of this game.

For a consistency matter, the participation rate that lenders take as given should coincide with the aggregation of their individual decision rules:

$$\tilde{PR}^R\left(\tilde{h}, b, y\right) = \int_i a_i\left(\tilde{h}, \tilde{PR}^R\left(\tilde{h}, b, y\right), b, y\right) dI\left(i\right) \tag{2.15}$$

And, finally, the participation rate after taking the CAC into account is:

$$\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right) \equiv \begin{cases} \tilde{PR}^R\left(\tilde{h}, b, y\right) & \text{if } \tilde{PR}^R\left(\tilde{h}, b, y\right) \in [0, T) \\ 1 & \text{otherwise} \end{cases} \tag{2.16}$$

Notice that $\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right)$ is the relevant function in the Nash bargaining process. During negotiations, the government and the lenders take as given not only the aggregation of lenders' individual decisions but also the enforcement of CACs.

2.2.4 Equilibrium

An equilibrium is a set of:

- value functions V, V^L, V^R, V^P, and V^D,
- government policy functions c^P, b^P, d^P, d^P_L, and d^P_R,
- lenders' individual decision rules a^P_i,
- participation rate functions for given restructuring offers before and after consideration of the CAC, $\tilde{PR}^R$ and $\tilde{PR}^R_{CAC}$, respectively,
- bond price functions q^P, q^D, and q^L, and
- renegotiaton rules h^R and PR^R,

such that:

1. given the renegotiaton rules, h^R and PR^R, and the bond price function, q^P, the government value and policy functions solve the government's problem,
2. given the bond price functions, q^D, the value functions, V^P and V^D, and the participation rate function for given restructuring offers after consideration of the CAC, $\tilde{PR}^R_{CAC}$, the renegotiation rules, h^R and PR^R, solve the Nash bargaining problem,
3. given the price function, q^L, the lenders' individual decision rule, a^P_i, solve the lender's individual problem,

4. given the lenders' individual decision rule, a_i^P, the participation rate function for given restructuring offers before the consideration of the CAC, $\tilde{PR}^R$, solve the fixed point problem defined in equation (2.15),

5. given the participation rate functions for given restructuring offers before the consideration of the CAC, $\tilde{PR}^R$, the analogous function after the consideration of the CAC, $\tilde{PR}^R_{CAC}$, is defined by equation (2.16),

6. and the bond prices are consistent with lenders making zero profits after adjusting for default risk.

Given the above definition, the price schedule needs to satisfy a few conditions. Next, I define the price of a bond held by non-holdouts when the government repaid its previous (non-holdout) lenders:

$$
\begin{aligned}
q^P\left(b', b_l, y\right) = & \frac{(1-\theta_L)}{1+r}\mathbb{E}_{y'|y}\left[1 - d^P\left(b', b_l, y'\right)\right] \\
& + \frac{(1-\theta_L)}{1+r}\mathbb{E}_{y'|y}\left[d^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \\
& + \frac{\theta_L}{1+r}\mathbb{E}_{y'|y}\left[1 - d_L^P\left(b', b_l, y'\right)\right] \\
& + \frac{\theta_L}{1+r}\mathbb{E}_{y'|y}\left[d_L^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right]
\end{aligned}
\tag{2.17}
$$

The first two lines are standard for most quantitative sovereign debt models with short term bonds and renegotiation, as they refer to the states in which litigation did not succeed. In the first one, the government chooses to repay (non-defaulted debt), $d^P\left(b', b_l, y\right) = 0$, while in the second one, the government chooses to default, $d^P\left(b', b_l, y\right) = 1$, in which case the lender holds a defaulted bond $q^D\left(b', y'\right)$. The remaining lines, on the other hand, refer to the states in which litigation succeeds. In the third one, all bondholders are paid in full, while in the fourth one, creditors hold a defaulted debt as litigation forces the government to default on all of them. Different than the early quantitative sovereign debt models, defaulted bonds generally feature strictly positive market price $q^D\left(b', y'\right)$, because they are not forgiven and are eventually restructured.

The price of a bond held by holdouts (or any lender who purchased the bonds from a holdout in secondary markets) is the following:

$$
\begin{aligned}
q^L\left(b', b_l, y\right) = & \frac{(1-\theta_L)}{1+r}\mathbb{E}_{y'|y}\left[\left[1-d^P\left(b', b_l, y'\right)\right] q^L\left(b^P\left(b', b_l, y'\right), b_l, y'\right)\right] \\
& + \frac{(1-\theta_L)}{1+r}\mathbb{E}_{y'|y}\left[d^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \\
& + \frac{\theta_L}{1+r}\mathbb{E}_{y'|y}\left[1-d_L^P\left(b', b_l, y'\right)\right] \\
& + \frac{\theta_L}{1+r}\mathbb{E}_{y'|y}\left[d_L^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right]
\end{aligned}
\tag{2.18}
$$

Its distinction from the bonds held by non-holdouts appears in the first line, where litigation fails and the holdouts continue to carry bonds priced at q^L. The third line is also worth mentioning: it indicates the holdouts successfully force the government to repay them in full.

The price of a debt in default is:

$$
\begin{aligned}
q^D\left(b, y\right) = & \frac{\theta_R}{1+r}\mathbb{E}_{y'|y}\left[\left[1-d_R^P\left(b, y'\right)\right] X\left(b, y'\right)\right] \\
& + \frac{\theta_R}{1+r}\mathbb{E}_{y'|y}\left[d_R^P\left(b, y'\right) q^D\left(b, y'\right)\right] \\
& + \frac{(1-\theta_R)}{1+r}\mathbb{E}_{y'|y}\left[q^D\left(b, y'\right)\right]
\end{aligned}
\tag{2.19}
$$

where

$$
X\left(b, y'\right) \equiv \max\left\{RR\left(b, y'\right), HO\left(b, y'\right)\right\} \tag{2.20}
$$

$$
RR\left(b, y'\right) \equiv 1 - h^R\left(b, y'\right) \tag{2.21}
$$

$$
HO\left(b, y'\right) \equiv \begin{cases} q^L\left(b_l^R\left(b, y\right), b_l^R\left(b, y\right), y'\right) & \text{if } PR^R\left(b, y'\right) < T \\ RR\left(b, y'\right) & \text{otherwise} \end{cases} \tag{2.22}
$$

Lenders continue to hold defaulted debt if the government rejects the restructuring terms or if no restructuring opportunities even arise, as captured by the second and third line of equation (2.19), respectively. The first line of equation (2.19), though, represents the lenders' payoff X from a successful renegotiation. This payoff is the maximum between the recovery rate on the unit of debt or the market value of a bond in litigation, subject to the CAC. Notice that, since the government has to remain in financial autarky during the period of restructuring, then the total stock of debt in the end of the period coincides with the debt held by holdouts, $b_l^R(b,y) \equiv \left[1 - PR^R(b,y)\right] b$[7].

2.3 Inspecting the mechanism

In this section, I discuss the endogenous participation rate, which is a novel feature of my model. I leave for section 2.4 to discuss some parameter-dependent properties of my model.

The participation rate is the outcome of the aggregation of individual decisions of atomistic lenders. They take the deal when the recovery rate is higher than the value of being a holdout, and reject the deal otherwise. Figure 2.1 helps visualize this aggregation problem. In each plot of this figure, I keep fixed the government's stock of debt in default and income. The vertical lines represent CAC thresholds. The horizontal lines represent a recovery rate $RR\left(\tilde{h}\right) \equiv 1 - \tilde{h}$ associated with some haircut $\tilde{h}$; notice that I fix a different haircut $\tilde{h}$ in each panel of figure 2.1. The dashed increasing curves are curve levels of equation (2.16), which represents the lender's payoff when the CAC is enforceable. Finally, the solid increasing curves are curve levels of $HO_1\left(\tilde{h}, \tilde{PR}, b, y\right) \equiv q^L\left(\left[1 - \tilde{PR}\right] b, \left[1 - \tilde{PR}\right] b, y\right)$, which represents the value of holding out of the restructuring when the CAC is not enforceable.

Notice from the solid increasing curve that the value of holding out (HO_1) becomes increasingly more valuable as the participation rate increases, ie $q^L\left(\left[1 - \tilde{PR}\right] b, \left[1 - \tilde{PR}\right] b, y\right)$

[7] Relaxing the assumption that the government remains in financial autarky during the renegotiation period is computationally challenging because the market value of a bond in litigation would be given by $q^L\left(b^P\left(b, b_l^R(b,y), y'\right), b_l^R(b,y), y'\right)$.

increases as $\tilde{PR}$ increases. The intuition behind this monotonicity is simple: holdouts free-ride on the debt relief provided by lenders who participate in debt restructurings. A successful litigation is more likely to trigger a new default episode if the amount of debt in dispute is higher; consequently, lenders have less incentives to become holdouts when only a small pool of bondholders accept the restructuring terms. Since the recovery rate is independent of the participation rate $\tilde{PR}$ and only depends on the haircut $\tilde{h}$, then the curves can intersect each other in, at most, one point[8].

The top left panel of figure 2.1 depicts the case in which the haircut $\tilde{h}$ offer is low enough to the point in which it's never advantageous to become a holdout. For any rate below 100%, holdouts have incentives to deviate from their strategy and take the deal. Thus, $\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right) = 100\%$ is the only rate that satisfies the consistency condition of equation (2.16).

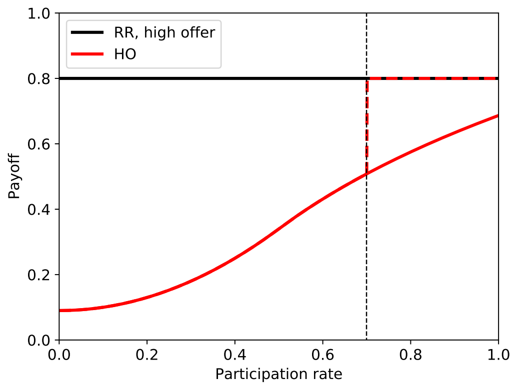
In the other extreme, the top right panel of figure 2.1 shows the case in which the haircut $\tilde{h}$ is high. In this case, no lender takes the deal ($\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right) = 0\%$) and, consequently, renegotiation fails.

This case of high haircut illustrates the purpose of introducing the lenders' belief of a negligible probability that the CAC is neglected. Consider an alternative framework in which lenders believe CACs are always enforceable, i.e., the CAC applies whenever the participation rate is above the threshold T. Then, two participation rates $\{0\%, 100\%\}$ become consistent with equation (2.16). When a lender takes as given that all other lenders are accepting the deal, it becomes indifferent between accepting or trying to holdout because the CAC will ensure that the payoff is $1-\tilde{h}$. This equilibrium with 100% participartion rate for such a high haircut is undesirable, since there are no forces driving any lender to accept the deal, except for the CAC itself. In my framework, though, where lenders believe CACs may be neglected, they never choose this weakly dominated strategy in equilibrium. The

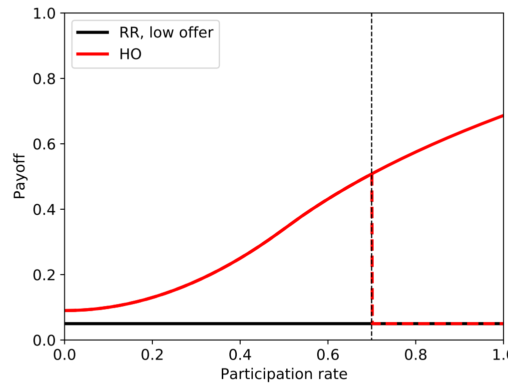
[8] Notice the recovery rate does not depend on anything but the haircut. This feature is due to the maturity of the debt. Since it is a one-period bond, then the government realizes a cash transfer to the lenders as soon as they take the deal. After that, participating lenders have no longer any relationship with government. The introduction of long term debt to this model would imply that the payoff of taking the deal is tied to the participation rate (as well as the government's stock of debt in default and income). In section 2.5, I discuss in further details the consequences of introducing long term bonds to this model.

introduction of CAC risk makes lenders reject the deal even if they believe that everybody else is taking the deal, because the tiny possibility of litigating makes the holdout strategy more valuable.

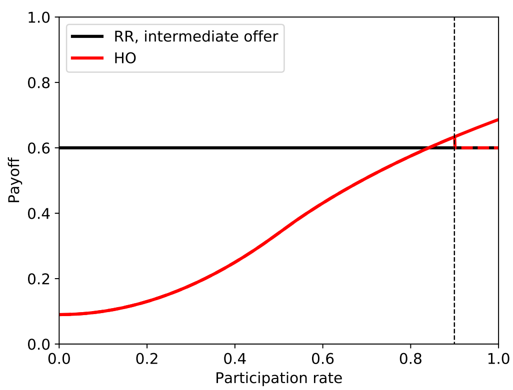
Figure 2.1: Lenders's payoff.



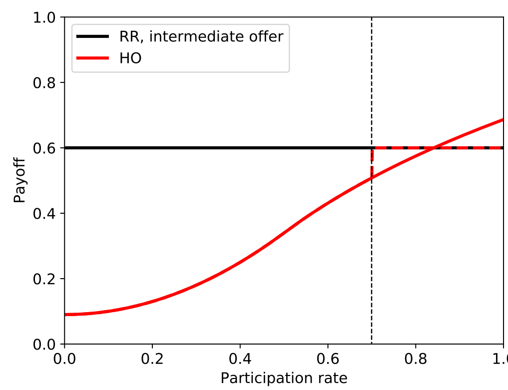
(a) High recovery rate.



(b) Low recovery rate.



(c) Intermediate recovery rate, loose CAC.



(d) Intermediate recovery rate, binding C.

Finally, the bottom panels of figure 2.1 consider the same intermediate haicut offer, but different CAC thresholds. In the bottom left panel, the CAC is too high and hence does not bind. As a consequence, the only outcome consistent with equation (2.16) is such that

$\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right) < T$, where $T \leq 100\%$[9]. On the other hand, in the bottom right panel, the CAC threshold is lower. In this case, the CAC binds and guarantees the government a full participation rate, $\tilde{PR}^R_{CAC}\left(\tilde{h}, b, y\right) = 100\%$.

2.4 Quantitative analysis

In this section, I describe the computational algorithm I used to solve my model and discuss some of its key features.

2.4.1 Computational algorithm

I numerically solve my model using value function iteration. I use discrete grids for the state and choice variables but use interpolation to find the participation rates.

1. Start with a guess for value functions, V, V^L, V^R, V^P, V^D, and price functions, q^P, q^L, q^D.
2. Solve for V^P, V^D, and b^P using the guesses.
3. Solve for the renegotiation outcomes h^R and PR^R using the guesses and the solution from step 2 (V^P and V^D).
4. Solve for V, V^L, V^R and d^P, d^P_L, d^P_R using the solution to step 2 (V^P and V^D) and to step 3 (h^R and PR^R).
5. Solve for q^P, q^L, q^D using the guesses for prices, the solution to step 2 (b^P), to step 3 (h^R and PR^R), and to step 4 (d^P, d^P_L and d^P_R)
6. Check for convergence of value functions and prices.
7. If no convergence, update guesses V, V^L, V^R, V^P, V^D, and price functions, q^P, q^L, q^D with the solution of the last iteration and repeat steps 2-7.

[9] A similar argument used when the haircut $\tilde{h}$ was high applies to this case of intermediate $\tilde{h}$ and loose CAC: the CAC risk eliminates the equilibrium with 100% participation rate.

2.4.2 Calibration

I consider the case of the Argentine debt crisis in 2001 for the calibration of my model and use the following functional forms. The income shock follows a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$, with $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. And I assume the direct output cost of default has a quadratic functional form $\phi(y_t) = \max\{0, \phi_0 y_t + \phi_1 y_t^2\}$, with $\phi_0 < 0 < \phi_1$, which makes default more costly during high-endowment periods. In particular, the cost is zero when for $0 \leq y_t \leq -\frac{\phi_0}{\phi_1}$ and increases more than proportionally for $y_t > -\frac{\phi_0}{\phi_1}$. This asymmetry allows the model to match default episodes occurring during bad times and, more generally, to better match the dynamics of spreads observed in the data[10]. I also assume that the utility function features a constant relative risk aversion (CRRA): $u(c_j) = \frac{c_j^{1-\eta}-1}{1-\eta}$ and that a period in the model corresponds to a quarter of a year.

I report in table 2.1 all the parameter values that can be directly calibrated from the data. The risk-free interest rate is set to 1%, the 1990-2001 average quarterly interest rate of a 5-year treasury bond[11]. The constant coefficient of relative risk aversion is set to a standard value, $\eta = 2$. Renegotiation opportunities arise with 2.7%, so that default episodes last 9 years, on average, while litigation succeeds with probability 5%, so that it is resolved in 5 years, on average[12]. As most debt contracts issued under the New York jurisdiction did not involve CACs before 2001, including those Argentine bonds, then I set the CAC threshold to 100% so that bondholders can provide debt relief only under unanimity. The AR(1) income process is estimated using HP-filtered logged Argentine GDP data from 1980 to 2001. This yields an auto-correlation parameter $\rho = 0.945$ and a standard deviation of innovations of $\sigma = 0.025$.

[10] See Arellano (2008), Chatterjee and Eyigungor (2012), and Hatchondo et al. (2014).

[11] I excluded the 1980s when computing the average of the US interest rate. Thus, I excluded the observations from the unusually high interest rates of the 1980s, when the then chairman of the Federal Reserve, Paul Volcker, raised interest rates to tame the exceptionally high US inflation.

[12] It took 4 years from the 2001 default episode until the first Argentine restructuring round, 9 years until the second round, and 14 years until the lawsuits "succeeded."

Table 2.1: Parameters directly calibrated from the data

Parameter		Value	Detail
Risk-free interest rate	r	0.010	1980-2001
Risk aversion	η	2.000	Standard
Prob(litigation)	θ_L	0.050	Duration of 5 years
Prob(renegotiation)	θ_R	0.027	Duration of 9 years
CAC threshold	T	1.000	No CAC
Income process	ρ_y	0.945	AR(1) estimation
	σ_y	0.025	

In table 2.2, I report the internally calibrated parameters: the discount factor β, the direct output cost parameters ϕ_0 and ϕ_1, and the lenders' bargaining power α. I set them to match four moments of the Argentine economy: the default probability of 3.0%, the average debt service-to-GDP ratio of 5.5%, the trade balance volatility relative to the GDP volatility of 17.1% and the average spread of 8.1%.

Table 2.2: Internally calibrated parameters and moments

Parameter		Value	Moment	Data	Model
Discount factor	β	0.943	Default probability	3.0%	3.1%
Bargaining power	α	0.103	Debt service-to-GDP (mean)	5.5%	5.0%
Output cost	d_1	-0.191	$\frac{\text{Trade balance (volatility)}}{\text{GDP (volatility)}}$	17.1%	24.7%
	d_2	0.246	Spread (mean)	8.1%	3.6%

2.4.3 Renegotiation outcome

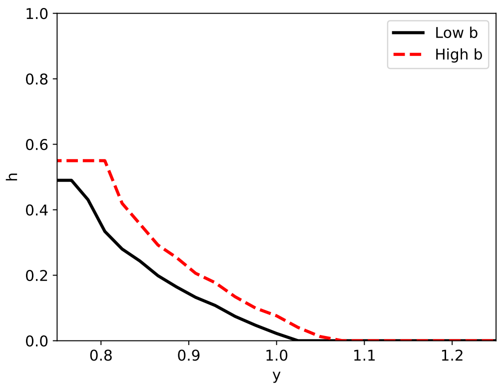
Consumption smoothing motives play an important role during restructuring episodes. A poor and financially distressed government in default receives reasonable debt relief during restructuring episodes. As a consequence, the government does not guarantee a full participation rate, as the lenders have incentives to become holdouts and start a litigation process that may last for many periods. Thus, the govenment incurs in only part of the restructuring costs in the current period, leaving part of it to the next periods, when it is likely to be in better times, given the mean reversion of the income shock. Precisely, in the current period, the government services only part of the stock of debt, with a discount, while in the future, when litigation suceeds, it may service the debt held by holdouts in full and face higher borrowing costs until then.

On the other hand, a rich country with low debt levels in default can afford a full participation rate by restructuring the debt with little or no haircut, which allows the country to prevent future costs, when the income of the country reverses downwards towards the mean. Figure 2.2 depict this dynamics[13].

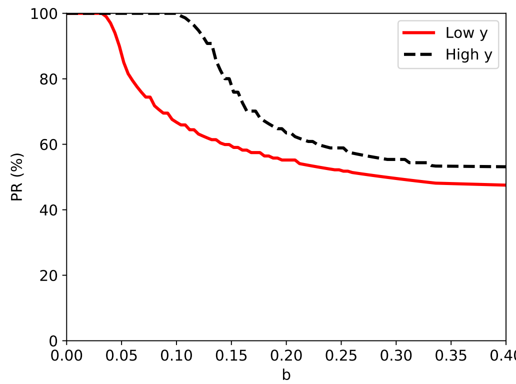
In the left panel of figure 2.2, the haircut is decreasing in the income level. For high enough income, the debt relief disappears and the government pays the full amount it owes; similarly, the debt relief also disappears if the governments had defaulted on a smaller amount of debt. In the right panel, the participation rate is increasing in the income level and decreasing in the debt level.

[13] There are other forces working in the same direction as the consumption smoothing motive. The asymmetric default cost $\phi(y_t)$, which is increasing in the income level, reduces the outside option of the government during the Nash Bargaining problem disproportionally more during periods of high income; as long as the lenders' bargaining power is strictly greater than zero, it leads them to claim more favorable restructuring terms. Also, since the income is persistent, this asymmetric cost also incentivizes the government to provide generous restructuring terms during good times, as it prevents litigation from pushing the country towards a new costly default in the near future.

Figure 2.2: Consumption smoothing dynamics.



(a) Haircut as a function of the income level.



(b) Participation rate as a function of the debt level.

Due to consumption smoothing motives, the states that in which renegotiations achieve lower participation rates are the states that feature higher debt relief. These results rationalize the empirical finding of Fang et al. (2021) in which haircuts and holdout rates are positively correlated.

2.4.4 The role of litigation

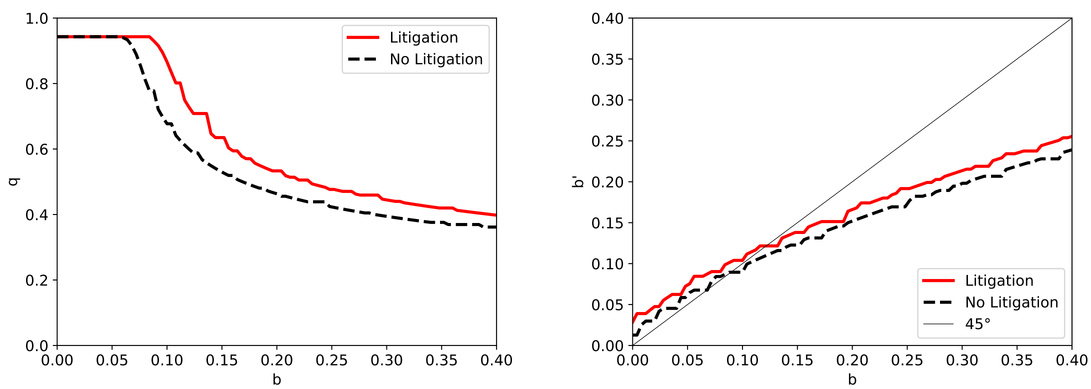
What distinguishes my paper from Yue (2010) is the possibility to hold out of restructuring deals. Thus, I evaluate the role of litigation in sovereign debt markets by comparing our models. In this comparison, her model captures a legal framework that prevents lenders from holding out and, to some extent, can be interpreted as an extreme "anti-vulture fund" legislation.

My model nests Yue (2010) if I set the probability of litigation success to zero, $\theta_L = 0$. The remaining parameters that I use are the same ones described in the tables 2.1 and 2.2. Figure 2.3 illustrates two important effects of litigation: it lowers the borrowing costs and drives the government to increase borrowing.

In the left panel of figure 2.3, I plot the bond price schedule in two different environments: the dashed curve refers to the environment without litigation while the solid curve refers to the case where lenders can become holdouts and litigate. For comparability reasons, I consider states whith no debt held by holdouts in the environment that allows litigation. Except for low enough borrowing choices in which the default probability is zero, bond prices are higher when the threat of litigation is present. Still with no debt held by holdouts, the right panel depicts the government's policy function in these two different environments. Notice the government borrows more when lenders have the ability to become holdouts.

In a sense, these figures provide an intuitive illustration to the role of litigation in sovereign debt markets: it buys commitment to the government's borrowing decisions.

Figure 2.3: The role of litigation in sovereign debt markets.



(a) Price schedule. (b) Borrowing decisions.

I also perform the following exercise. I consider three different economies and, for each of them, simulate 5,000 periods and drop the first 500. First, I simulate the litigation-free and the litigation-prone economies and compute their average spreads and debt-to-GDP ratios. I find that the debt-to-GDP is 0.02pp higher in the litigation-prone and spreads are virtually the same. Of course, spreads are sensitive to the debt accumulation in these two different environments. Thus, I also consider an alternative setting. I simulate the litigation-free economy using the price schedule of the litigation-prone economy. In this alternative economy, spreads are 0.21pp lower than in the litigation-free economy, even though both share the same borrowing decision rule.

2.4.5 The role of collective action clauses

In this section, I consider the design of debt contracts for the Argentine economy. To quantify the welfare gains of transitioning away from debt contracts that require unanimous decisions for changing payment terms, I solve my model for many economies that share the same set of parameters described in the tables 2.1 and 2.2, except for the CAC threshold T. Then, I proceed as follows. First, I depart from the ergodic distribution of an economy that lacks CACs and simulate 2,000 periods of an economy with a CAC threshold T. Then, I compute its associated consumption path in these 2,000 periods and search for the optimal threshold T by varying it from 0% to 100% in jumps of 5pp. For each T, I repeat this procedure 200 times.

Table 2.3 summarizes the main findings. I report the welfare gain relative to an economy that continued to issue bonds with no CACs. Precisely, I compute the consumption compensation that would make the government indifferent to including CACs to its debt contracts. The optimal clause has a threshold $T = 80\%$, and renders a welfare gain of 0.09%. This is not too far from the typical threshold present in sovereign debt contracts, $T = 75\%$, documented by Bradley and Gulati (2014), in which the welfare gain is only 0.02pp below the optimal contract.

Table 2.3: Welfare gains

Threshold T	Welfare gain
75%	0.07%
80%	0.09%
100%	0.00%

2.5 Conclusion

In this paper, I study the role of litigation and collective action clauses in sovereign debt markets. By introducing a holdout problem to an otherwise standard quantitative model, I endogenize the participation rate in sovereign debt restructuring episodes. I show that, on one hand, litigation buys commitment to the government and hence facilitates borrowing; on the other hand, I show that CACs can mitigate the holdout problem that litigation introduces. Finally, the main findings of this chapter are that, through the lens of my model, "anti-vulture fund" legislation can be detrimental to debtor countries and the optimal CAC for Argentina has an 80% threshold, which is not far from the widespread threshold currently observed in sovereign debt markets.

The model has some limitations. Here I highlight one of them: cash transfers from renegotiation. Further exploration of my model may introduce longer debt maturities or debt restructurings with grace periods. These changes would ensure that the payoffs of creditors who takes a restructuring deal remain somewhat tied to the payoffs of the government even after the restructuring date. This setup would reinforce the holdout problem as it introduces a new negative externality that stems from litigation, since holdouts can lead the government to default. Thus, litigation would impose costs on creditors negotiating debt relief, not only on future creditors.

Future exploration of my model may also investigate how the optimal regulation (or CAC) depends on the countries' institutional quality, especially the Heavily Indebted Poor Countries (HIPC). Countries with poor institutions, that are susceptible to more impatient governments with very short-term goals, may not benefit from litigation as other developing countries. Since their governments already borrow more than what their households prefer, litigation may drive them to further overborrow.

Chapter 3

Reserve Management During Default Episodes

3.1 Introduction

Why do countries accumulate foreign reserves? The forces against a simultaneous accumulation of foreign reserves and external debt are very obvious: economies facing default risk pay high interest rates on their liabilities and receive low interest rates on their assets. Yet, this joint accumulation of reserves is the general patterns observed in most countries. In this chapter, I revisit this question and introduce a new motive for countries to accumulate reserves: I show that reserves may increase the government's debt relief by improving the their bargaining position. Although there is a large and growing literature on reserves accumulation, studies have rarely addressed the interaction between restructuring processes and foreign reserves.

This paper contributes to two different strands of the sovereign default literature that follows the seminal work by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008): management of foreign reserves and debt renegotiation.

Alfaro and Kanczuk (2009), which is closely related to this paper, enhances a sovereign debt model to incorporate the possibility that the government accumulates foreign assets.

They find that the optimal policy is to hold no reserves at all. In their setting, the main motive to hold reserves is to transfer resources from the good times in which the goverment has access to international financial markets to periods in which the government is in default and unable to borrow. In contrast to their paper, I explore a bargaining motive.

Other models have been more successful in terms of rationalizing simultaneous holdings of debt and reserves. They have departed from the simple off-the-shelf sovereign debt model of Alfaro and Kanczuk (2009) in which both reserves and debt share the same one period maturity and income shocks are the only source of uncertainty, and introduced a series of changes.

A few papers show that reserves are useful to get through sudden stops. Hur and Kondo (2016) shed light on the upward trend in the reserves-to-debt ratio by studying the accumulation of reserves in a multi-country model with endogenous sudden stops. Bianchi et al. (2018) show that, when the maturity of debt is longer than the maturity of savings, reserves provide a hedge against rollover risk. Tavares (2015) studies the role of international reserves in sovereign debt restructuring under fiscal adjustment. In his framework, reserves bring insurance benefits against default and rollover risk. In particular, reserves reduce the need for distortionary taxation. Arce et al. (2019) provides a macroprudential theory of reserve accumulation by showing that the government accumulates international reserves to reduce the exposure to sudden stops due to overborrowing by the private sector.

Bianchi and Sosa-Padilla (2020) show the accumulation of reserves may provide macroeconomic stability for governments that face nominal rigidities and follows a fixed exchange rate regime. Particularly, issuing (long term) debt to accumulate (short term) reserves may allow the government to reduce the average and volatility of unemployment.

Samano (2021) provides a theory of reserve accumulation that emphasizes the role on an independent central bank in an environment in which the government lacks fiscal discipline. His model departs from the standard assumption that the government behaves as a consolidated entity choosing reserves and debt simultaneously. Instead, it considers two policymakers: a central bank that manages foreign reserves and a government that issues public debt. The dynamics is that the fiscal authority overborrows, and so the monetary authority accumulates reserves in an attempt to undo the overborrowing.

Next, in Section 3.2, I describe the model and, in Section 3.3, I describe the mechanism. Then, in Section 3.4, I discuss the numerical exercise. Finally, I conclude the paper in Section 3.5.

3.2 Model

I study optimal reserve management and debt renegotiation in a dynamic model of a small open economy. The government inherits a stock of debt in default b that is owed to a large number of risk averse foreign lenders. The domestic economy suffers a direct output cost and stays in autarky until the debt is restructured.

There are two periods: $t \in \{1, 2\}$. The government is benevolent. Households observe their endowment y_1 in the first period and receive an stochastic endowment of a tradable good y_2 in the final period. This process follows a finite-state Markov chain with transition probabilities $Prob\,(y_2 = y'|y_1 = y) = F\,(y'|y)$. Households are identical, with preferences given by:

$$E_t \left[\sum_{j=t}^{2} \beta^{j-t} u\,(c_j) \right] \tag{3.1}$$

where $\beta \in (0, 1)$ is the subjective discount factor, c is consumption, and the utility function u is strictly increasing and strictly concave.

In period 1, the government is in default and observes the total stock of defaulted debt b, stock of foreign reserves a, and income y. The domestic country suffers a direct output cost $\phi\,(y)$, which is increasing in income y. The government's debt is frozen: debt service is suspended and the government cannot borrow. The only policy instrument available is trading foreign reserves. The government's problem in period 1 is:

$$V^D\,(b, a, y) \equiv \max_{a'} u \left(y + a - \frac{a'}{1+r} \right) + \beta \mathbb{E} \left[V^R \left(b, a', y' \right) \right] \tag{3.2}$$

Notice that the government anticipates a renegotiation opportunigy in the subsequent (and final) period. A restructured debt characterizes the outcome of this renegotiation, and the government can take or reject this deal. Thus, the government faces the following problem in the second period:

$$V^R(b,a,y) \equiv \max_{d\in\{0,1\}} \left\{(1-d)\, V^P\left(b^R(b,a,y), a, y\right) + dV^A(a,y)\right\} \tag{3.3}$$

where $b^R(b,y)$ is the new total stock of debt after a successful debt restructuring and d is the default decision: $d = 1$ if the government rejects the deal with lenders and $d = 0$ otherwise.

The value of rejecting the deal is:

$$V^A(a,y) \equiv u(h(y) + a) \tag{3.4}$$

where $h(y) = y - \phi(y)$ is the endowment net of default costs. And the value of taking the deal is:

$$V^P(b,a,y) \equiv u(y - b + a) \tag{3.5}$$

By accepting the renegotiation terms, notice that the government avoids the direct output cost $\phi(y)$. Since period 2 is the last period, there is no room for further borrowing or savings. Anticipating this trivial equilibrium outcome, I omitted the policy functions for debt and reserves accumulation in the final period.

The solution to the government's problems gives decision rules for asset holdings, $a^P(b,a,y)$, and restructuring, $d^P(b,a,y)$. Next, I discuss some properties of these decision rules as a function of a bargaining process that drives the renegotiation outcome. And, finally, I discuss how the model behaves under a Nash Bargaining process.

3.3 Mechanism

3.3.1 Reserve Management

In this section, I consider an efficient bargaining process that always lead the parties to agree on some debt restructuring terms. From the first order conditions:

$$\frac{u'\left(y+a-\frac{a'}{1+r}\right)}{1+r}=\beta\mathbb{E}\left[\frac{\partial V^{R}\left(b,a',y'\right)}{\partial a'}\right] \tag{3.6}$$

Given the efficiency of the bargaining process, then $\frac{\partial V^R(b,a',y')}{\partial a'}=\frac{\partial V^P\left(b^R(b,a',y'),a',y'\right)}{\partial a'}$. Then, the FOC can be rewritten as:

$$u'\left(y+a-\frac{a'}{1+r}\right)=\beta\left(1+r\right)\mathbb{E}\left[u'\left(y'-b^{R}\left(b,a',y'\right)+a'\right)\left[1-\underbrace{\frac{\partial b^{R}\left(b,a',y'\right)}{\partial a'}}_{\text{bargaining mechanism}}\right]\right] \tag{3.7}$$

The standard consumption smoothing motive applies here. If the government has low endowment, then the marginal utility in the current period is low. Given the mean reversing property of the endowment process, the marginal utility in the subsequent period is likely to be higher than in the current period. Therefore, the government has incentives to save.

Nevertheless, renegotiation introduces the last term in equation 3.7, which may reinforce this incentive to accumulate reserves. The government has the ability to transform $\frac{1}{1+r}$ units of consumption in the initial period into $1-\frac{\partial b^R(b,a',y')}{\partial a'}$ units of consumption in the final period instead of just 1 unit. Therefore, it strengthens the incentive to save only if the renegotiated debt is decreasing in the asset position, $\frac{\partial b^R(b,a',y')}{\partial a'}<0$.

3.3.2 Renegotiation

Nash Bargaining

Nash bargaining is the most common cooperative process used in the literature of quantitative sovereign debt models. Next, I show that the reserve dyamics that this process delivers heavily depends on the shape of the utility function.

In the final period, lenders and the government have an opportunity to restructure the debt. They observe the amount of debt in default b and the income shock y, and then negotiate a new restructured debt $\tilde{b}$. The renegotiated debt is the solution to the following Nash Bargaining problem:

$$
\begin{aligned}
b^R(b,a,y) = \arg\max_{\tilde{b}\in[0,b]} & \left\{ S^{LEN}\left(\tilde{b}\right)^{\alpha} S^{GOV}\left(\tilde{b},a,y\right)^{1-\alpha} \right\} \\
s.t. \quad S^{LEN}\left(\tilde{b}\right), & \quad S^{GOV}\left(\tilde{b},a,y\right) \geq 0
\end{aligned} \tag{3.8}
$$

where S^{GOV} is the government's surplus; α and S^{LEN} are the lenders' bargaining power and surplus, respectively. As in chapter 2, renegotiation only succeeds if both parties voluntarily take the deal, i.e., if both, the government and lenders, are better off with the new restructured debt.

Since this renegotiation takes place in the terminal period, a failure implies that defaulted debt is never restructured. Hence, lenders' outside option is zero, and their surplus is:

$$
S^{LEN}\left(\tilde{b}\right) \equiv \tilde{b} \tag{3.9}
$$

The government's surplus is analogously defined as the difference between the value of accepting the deal and the value of remaining in default. Formally:

$$
S^{GOV}\left(\tilde{b},a,y\right) \equiv V^P\left(\tilde{b},a,y\right) - V^A(a,y) \tag{3.10}
$$

The first order condition is helpful to understand the forces at play in a Nash Bargaining setup. It states that the government-lenders' surplus ratio is inversely proportional to the ratio of their bargaining power and their marginal surpluses (see derivation in appendix A).

$$
\frac{S^{GOV}\left(\tilde{b},a,y\right)}{S^{LEN}\left(\tilde{b}\right)} = -\frac{(1-\alpha)}{\alpha}\frac{S^{GOV}_{\tilde{b}}\left(\tilde{b},a,y\right)}{S^{LEN}_{\tilde{b}}\left(\tilde{b}\right)} \tag{3.11}
$$

Improving the outside option. One force stems from higher reserves a improving the outside option $V^A(a,y)$ relatively more than the value of taking the deal $V^P\left(\tilde{b},a,y\right)$), which drives the restructuring terms to be more favorable to the government. The RHS of equation (3.11) provides a good illustration: notice the denominator $S^{LEN}\left(\tilde{b}\right) \equiv \tilde{b}$ does not depend on the asset position. And since the government's surplus has to be positive and the utility function is increasing and concave, then the numerator is increasing in a. Therefore, to keep the RHS constant, increases in a lead to reductions in $\tilde{b}$.

Decreasing returns. But there is another force in the opposite direction: at the margin, wealthier government enjoys less benefits from debt reliefs. And this is captured by the LHS of equation (3.11). Again, notice that the denominator does not depend on reserves while the numerator does. The derivative of the government's surplus is $V_{\tilde{b}}^{P}\left(\tilde{b}, a, y\right)$. Since the marginal utility is decreasing in consumption, then the numerator is decreasing in reserves. Therefore, to keep the LHS constant, increases in $\tilde{b}$ has to compensate any increases in a.

Nash Bargaining - Alternative Utility Function

The first force (improving the outside option) does not prevail for a wide range of parameters when using Nash bargaining with, for instance, Constant Relative Risk Aversion (CRRA) utility function. Indeed, in my numerical exercises, I show that reserve accumulation has a meaningless effect on restructuring outcomes.

In this section, I follow Pitchford and Wright (2016) and turn to an alternative utility function: $u(y, c, z) = m(\Psi(y, z))\, c$, where m is positive and decreasing, and $\Psi(y, z) \equiv zh(y) + (1-z)\, y$ and z indicates whether the country is in default. It features much of the simplicity of linear preferences while avoiding model internal inconsistencies[1].

With this functional form, I obtain in a closed-form solution how building up reserves while in default affects the debt restructuring outcome (see derivation in appendix A):

$$\frac{\delta b^{R}(b, a, y)}{\delta a} = \alpha \left[\frac{m(y) - m(h(y))}{m(y)}\right] < 0 \tag{3.12}$$

[1] Notice that a risk neutral government would not be in default in the first place, especially if lenders and the government discount the future at the same rate (indeed, I use $\beta(1+r) = 1$ in the next section's numerical exercice). Furthermore, even if I assumed a risk neutral government in default, reserves accumulation would still have no effect on restructuring outcomes. To see this, notice from the government's renegotiation surplus that reserves in repayment would cancel out with reserves in default.

No matter what is the state of nature, higher reserves improves the government's renegotiation terms. There are two reason for this result. As in the general case, reserves disproportionally increase the government's outside option, since the marginal utility in default $m(h(y))$ is higher than in repayment $m(y)$[2]. And given the preferences linearity with respect to reserves, wealthier governments do no feature lower marginal utility.

Modified Nash Bargaining

Next, instead of choosing a particular utility function, I explore an alternative process that is identical to the Nash bargaining previously presented, except that the lenders' bargaining power is not a constant but a decreasing function of the government's asset position: $\alpha(a) \equiv \alpha \frac{(A-a)}{A}$. Thus, as the government accumulates more reserves, the weight of its surplus in the government-lenders joint maximization also increases[3]. This specification ensures that, for a wide range of parameters, the forces pushing toward a negative relationship between restructured debt and reserves prevail.

3.4 Numerical Exercise

In this section, I consider the standard and modified Nash bargaining processes. I use the following functional forms. The income shock follows a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$, with $|\rho| < 1$ and $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$. And I assume the direct output cost of default has a quadratic functional form $\phi(y_t) = \max\left\{0, \phi_1 y_t + \phi_2 y_t^2\right\}$, with $\phi_0 < 0 < \phi_1$. Finally, I also assume that the utility function features a constant relative risk aversion (CRRA): $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$.

[2] I assume that $\frac{m(y)(y+a)}{m(h(y))[h(y)+a]} > 1$, for any value of y and a. To some extent, it means that the marginal utility is not too sensitive to the income. Otherwise, there would be states in which the government would actually enjoy being in default, and the government would reject any deal, including a 100% debt forgiveness.

[3] In my computational solution, I set A equals to 0.4, which is also the highest value that reserves can assume. Therefore, from the modified function for the lenders' bargaining power: if the government has no assets, then its weight is zero while; similarly, if the government has the maximum amount of reserves, 0.4, then its weight is 1

In table 3.1, I report all the parameter values I use in the numerical exercise.

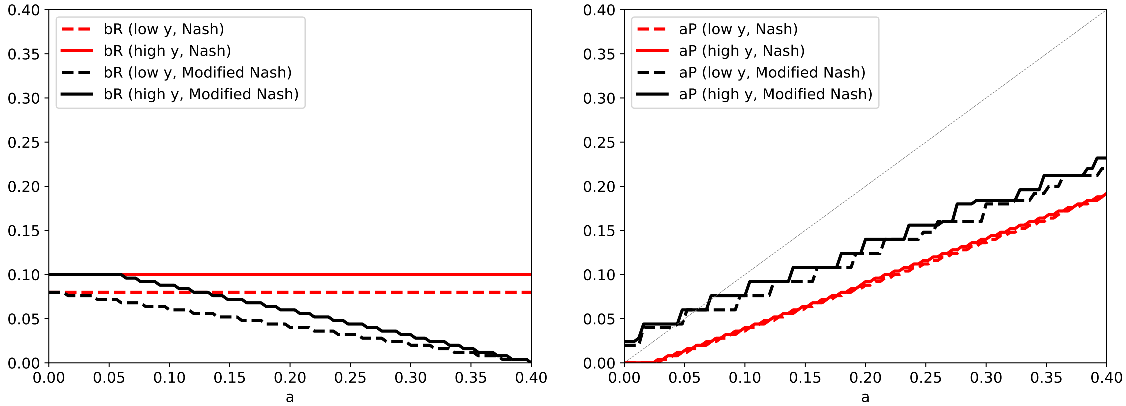
Table 3.1: Parameters

Parameter		Value
Risk-free interest rate	r	0.015
Discount factor	β	0.985
Bargaining power	α	0.800
Risk aversion	η	2.000
Income process	ρ_y	0.945
	σ_y	0.025
Output cost	ϕ_1	-0.128
	ϕ_2	0.245

I show the main results in figure 3.1. Notice that, under Nash bargaining, the renegotiation outcome does not depend on reserves. On the other hand, the modified Nash bargaining makes the debt relief more generous when the country is wealthier. As a consequence, in the initial period, the government saves more in the modified than standard Nash bargaining setting. The government has not only the standard consumption smoothing motive guiding its savings decision, but also the intention to build up its bargaining power during renegotiation efforts[4].

[4] Notice from table 3.1 that impatience is not driving the savings decision, as the government is as patient as the foreign lenders.

Figure 3.1: Reserves management: Nash bargaining vs modified Nash bargaining



(a) Renegotiated debt.

(b) Savings policy.

3.5 Conclusion

In this paper, I study the interaction between savings decisions and renegotiation outcomes. In particular, under specific functional forms of preferences and restructuring processes, I show that countries can use reserves to improve their bargaining position. Consequently, countries have incentives to build up their reserves during default episodes.

I intend to expand this work with an infinite horizon version of my model. Building up reserves during default times would reduce the price of bonds when countries are not in financial autarky[5]. This costly consequence may be addressed with restrictions to the savings policy to minimize a potential "over-savings during default" problem.

[5]Notice that building up reserves before default may still improve the government's borrowing conditions, as the increase in the government's ability to repay may still outweight the negative effect of potentially bargaining with more reserves following an eventual default.

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Appendix A

Chapter 3

I show in chapter 3 that, under Nash bargaining, the government-lenders' surplus ratio is inversely proportional to the ratio of their bargaining power and their marginal surpluses. I provide the derivation in this appendix.

The FOC from the Nash bargaining problem is the following:

$$\alpha S_{\tilde{b}}^{LEN}\left(\tilde{b}\right)\left(\frac{S^{GOV}\left(\tilde{b},a,y\right)}{S^{LEN}\left(\tilde{b}\right)}\right)^{1-\alpha}+\left(1-\alpha\right)\left(\frac{S^{GOV}\left(\tilde{b},a,y\right)}{S^{LEN}\left(\tilde{b}\right)}\right)^{-\alpha}S_{\tilde{b}}^{GOV}\left(\tilde{b},a,y\right)=0 \tag{A.1}$$

Multiply both sides by $\left(\frac{S^{GOV}\left(\tilde{b},a,y\right)}{S^{LEN}\left(\tilde{b}\right)}\right)^{\alpha}$:

$$\alpha S_{\tilde{b}}^{LEN}\left(\tilde{b}\right)\frac{S^{GOV}\left(\tilde{b},a,y\right)}{S^{LEN}\left(\tilde{b}\right)}+\left(1-\alpha\right)S_{\tilde{b}}^{GOV}\left(\tilde{b},a,y\right)=0 \tag{A.2}$$

And rearrange it:

$$\frac{S^{LEN}\left(b^{R}\left(b,a,y\right)\right)}{S^{GOV}\left(b^{R}\left(b,a,y\right),a,y\right)}=-\frac{\alpha}{\left(1-\alpha\right)}\frac{S_{b}^{LEN}\left(b^{R}\left(b,a,y\right)\right)}{S_{b}^{GOV}\left(b^{R}\left(b,a,y\right),a,y\right)} \tag{A.3}$$

Now, I turn to a specific functional form of utility function that resembles linear preferences. Due to its simplicity, I show in chapter 3 a closed-form solution for how building up reserves while in default affects the debt restructuring outcome. Next, I proceed with the derivation.

With the alternative utility function $u(y,c,z) = m(\Psi(y,z))\,c$, this optimality condition can be rewritten in the following way:

$$\frac{b^R(b,a,y)}{m(y)\left(y - b^R(b,a,y) + a\right) - m(h(y))\left[h(y) + a\right]} = -\frac{\alpha}{(1-\alpha)}\frac{1}{\left[-m(y)\right]} \tag{A.4}$$

Take the derivative with respect to foreign reserves:

$$\frac{\frac{\partial b^R(b,a,y)}{\partial a} S^{GOV}\left(b^R(b,a,y),a,y\right) - b^R(b,a,y)\left[m(y)\left[1 - \frac{\partial b^R(b,a,y)}{\partial a}\right] - m(h(y))\right]}{\left[S^{GOV}\left(b^R(b,a,y),a,y\right)\right]^2} = 0 \tag{A.5}$$

From the RHS, notice that the lenders-government ratios of bargaining power and marginal surpluses do not vary with reserves. But, from the LHS, the ratio of their surpluses does change with reserves.

Next, multiply both sides by $\left[S^{GOV}\left(b^R(b,a,y),a,y\right)\right]^2$ and rearrange it:

$$\frac{\partial b^R(b,a,y)}{\partial a} = \frac{b^R(b,a,y)\left[m(y) - m(h(y))\right]}{\left[S^{GOV}\left(b^R(b,a,y),a,y\right) + b^R(b,a,y)\,m(y)\right]} \tag{A.6}$$

From the optimality condition, notice that $b^R(b,a,y) \equiv \frac{\alpha}{(1-\alpha)}\frac{S^{GOV}\left(b^R(b,a,y),a,y\right)}{m(y)}$:

$$\frac{\partial b^R(b,a,y)}{\partial a} = \frac{\frac{\alpha}{(1-\alpha)}\frac{S^{GOV}\left(b^R(b,a,y),a,y\right)}{m(y)}\left[m(y) - m(h(y))\right]}{\left[S^{GOV}\left(b^R(b,a,y),a,y\right) + \frac{\alpha}{(1-\alpha)}\frac{S^{GOV}\left(b^R(b,a,y),a,y\right)}{m(y)}m(y)\right]} \tag{A.7}$$

Rearrange it:

$$\frac{\partial b^R(b,a,y)}{\partial a} = \alpha\frac{\left[m(y) - m(h(y))\right]}{m(y)} < 0 \tag{A.8}$$

which is negative since m is a decreasing function and $h(y) < y$.